JOURNAL OF APPLIED POLYMER SCIENCE VOL. 8, PP. 455-466 (1964)

# Cut Growth and Fatigue of Rubbers. I. The Relationship between Cut Growth and Fatigue

A. N. GENT,* P. B. LINDLEY, and A. G. THOMAS, *The Natural Rubber Producers' Research Association, Welwyn Garden City, Herts., England*

## Synopsis

The relationship between the cut growth and fatigue failure of natural rubber vulcanizates under repeated loading is examined. The cut growth behavior has been investigated using several types of test piece, and the results are shown to be consistent when interpreted in terms of the tearing energy concept developed previously. The most comprehensive data have been obtained by measuring the growth of a small cut in the edge of a strip cycled in simple extension. It is found that the cut growth per cycle is approximately proportional to the square of the maximum tearing energy attained during the cycle. Using this relation, the fatigue life of a specimen containing a small cut is deduced from elasticity theory as a function of initial cut size and maximum strain. Experimental results give good agreement with theory. A similar strain dependence is found for the fatigue life of die-stamped dumbbell test pieces with no intentionally produced flaws; this is consistent with the mechanism of failure being cut growth from small flaws present in the specimens. Their effective size is estimated to be about $2 \times 10^{-3}$ cm, which is compatible with the observed superficial imperfections of the cut edges.

## Introduction

Of the many possible causes of failure of vulcanized rubbers, one particular process is considered here, termed "fatigue" failure. It consists of the gradual weakening of rubber specimens and eventual fracture brought about by repeated deformations much lower than the breaking strain. It is attributed in the present study to the growth by repeated tearing of a small flaw, or number of flaws, in the test piece surface. A treatment for the mechanics of failure by such a process is developed for rubber strips subjected to repeated extensions and an experimental examination of the principal theoretical predictions is described for a natural rubber vulcanizate.

Another form of deterioration has sometimes been referred to as fatigue; it is associated with the rise in temperature (heat build-up), caused by rapidly repeated deformations and appears to be primarily due to chemical effects. Indeed, similar changes can be achieved by the application of

* Present address: Rubber Research Institute, University of Akron, Akron, Ohio.

heat, without imposing any deformation. In the present experiments deterioration due to this cause was avoided. The test pieces were in the form of thin strips of large surface area, and the deformations were imposed at relatively low frequencies, so that the temperature rise of the test piece was insignificant.

## Mechanics of Cut Growth

When a test piece of gum natural rubber containing a sharp cut is slowly stretched, tearing occurs at the tip of the cut. Initially, tearing continues only as long as the deformation is increasing and ceases as soon as the deformation is held constant. This tearing is called smooth cut growth, from the observed smooth nature of the torn surface. The amount of cut growth continues to increase with increase in extension until, suddenly, there is a rapid increase in the length of the cut. This is called catastrophic tearing.

Rivlin and Thomas[1] have shown that the growth of a cut by catastrophic tearing is described by an energy criterion. Thomas[2] has shown further that the small growth of a cut $\Delta c$ before catastrophic tearing occurs is also determined by the energy $T$ available for cut propagation, the following approximate empirical relationship being obeyed:

$$\Delta c = T^2/G_s \qquad (1)$$

where $G_s$ is a constant, described hereafter as the smooth cut growth constant of the rubber. The tearing energy $T$ is given by

$$T = -(1/t)(\partial U/\partial c)_l \qquad (2)$$

where $t$ is the thickness of the test piece, $l$ its deformed length, $c$ the length of the cut, and $U$ is the energy stored elastically by the imposed deformation.

When the test piece is repeatedly deformed to a given $T$ value below the characteristic value $T_c$ at which the cut grows catastrophically and relaxed, the cut grows by a small amount each cycle. With the cut made by a razor blade it is found that, initially, the average amount of growth per cycle is given by eq. (1). However, after some hundreds of cycles, a gradual reduction in the rate of growth occurs, and finally, after a few thousand cycles it becomes substantially constant. This decrease in rate is accompanied by a visible roughening of the tip of the cut. From experiments of this nature it was found that a relationship similar to eq. (1) is applicable to rough cut growth, viz.,

$$dc/dn = T^2/G \qquad (3)$$

where $G$ is the rough cut growth constant of the rubber. The ratio of the two cut growth constants, $G/G_s$, was about 10:1.[2]

When expressed in terms of $T$, the cut growth behavior, like other rupture phenomena, is independent of the form of the test piece used in the experiments. Thus eqs. (1) and (3) describe intrinsic properties of the material.

### Evaluation of *T*

The evaluation from measured forces or strains of the tearing energy $T$, as defined in eq. (2), is possible only for certain types of test piece. Two of these, the "trousers" and pure shear test pieces, are briefly referred to in a later section; they are illustrated in Figures 1 and 2, respectively.

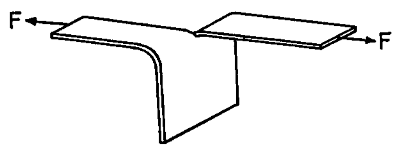


Fig. 1. Trousers test piece.

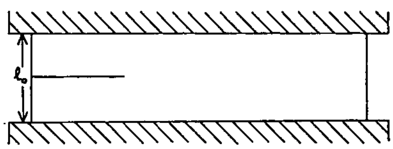


Fig. 2. Pure shear test piece.

The tearing energy is determined from the measured applied forces and strains, the relevant expression for the trousers test piece being[1]

$$T = (2\lambda F/t) - wW$$

where $w$ and $t$ are the total width and thickness of the test piece, respectively, $W$ the strain energy per unit volume in the legs of the test piece, $\lambda$ their extension ratio, and $F$ the applied force. The values of $w$ are normally chosen so that $\lambda \simeq 1$, when

$$T \simeq 2F/t$$

For the pure shear test piece[1]

$$T = Wl_0$$

where $l_0$ is the unstrained length and $W$ the strain energy per unit volume in that region of the test piece which is in a state of pure shear.

Most of the experimental work referred to in later sections has been carried out on a tensile strip (Fig. 3) for which the tearing energy is given by[1]

$$T = 2kWc \tag{4}$$

where $W$ is the strain energy per unit volume in the regions of the test piece remote from the cut, $c$ is the length of the cut, and $k$ is a function of

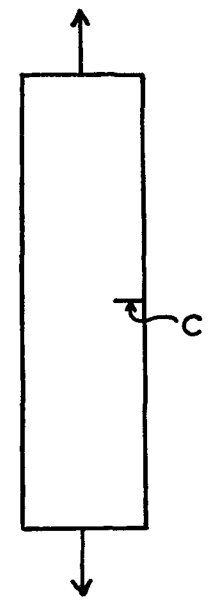


Fig. 3. Tensile strip.

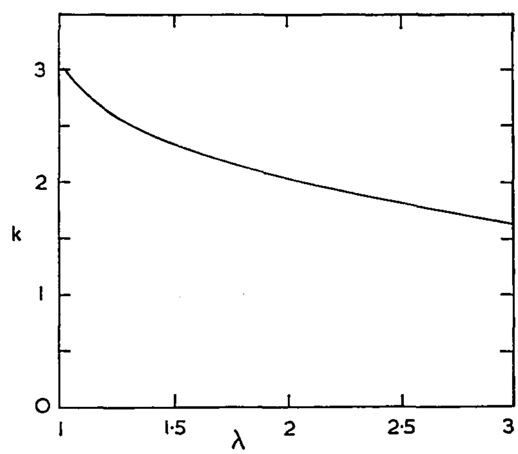


Fig. 4. Relationship between $k$ and $\lambda$ from the experimental data of Greensmith.[3]

the extension ratio $\lambda$. The manner in which $k$ depends on $\lambda$ can, in principle, be solved mathematically, but in practice this problem proves intractable. Greensmith,[3] however, has experimentally determined $k$ at various values of $\lambda$ by purely elastic measurements, and his results are shown in Figure 4. At extension ratios of 2–3, $k$ is about 2 and not very sensitive to strain. It is worth noting that a value of 2 was found necessary[1] to correlate tear measurements on a test piece of this form with those on a trousers type.

## Analysis of Fatigue

Failure in fatigue is observed to occur by the growth of a crack through the test piece, and this suggests that the fatigue process may be essentially cut growth from flaws or stress raisers initially present in the rubber. A theory of the fatigue failure of tensile strips is developed below on this basis.

When tensile strip containing a cut of length $c$ (Fig. 3) is cycled from its unstrained state, the cut growth rate will be given by

$$dc/dn = (2kW)^2c^2/G \tag{5}$$

from eqs. (3) and (4), where $n$ is the number of cycles and $W$ is the strain energy density at the maximum extension ratio $\lambda$ occurring during each cycle.

The number of cycles $n$ required to cause the cut to grow from an initial length $c_0$ to a length $c$, obtained by integrating eq. (5), is

$$n = [G/(2kW)^2][(1/c_0) - (1/c)] \tag{6}$$

This equation shows that $n$ becomes independent of $c$ when the latter is much greater than $c_0$, and that a finite number of cycles is necessary to cause an indefinite increase in cut length. Thus, according to the above theory, the size of a specimen will have little effect on the number of cycles to failure provided its dimensions are much greater than $c_0$. The number of cycles to break $N$ (i.e., the fatigue life) will therefore be given from eq. (6), with $c >> c_0$, by

$$N = \frac{G}{(2kW)^2c_0} \tag{7}$$

## Experimental Methods

The test pieces used in the experimental work were cut from molded sheets, about 1 mm. thick, of the following composition by weight: natural rubber (RSS1) 100, zinc oxide 5, stearic acid 2, sulfur 2.5, $n$-cyclohexyl benzthiazyl sulfenamide 0.6, phenyl-$\beta$-naphthylamine 1; vulcanization was for 40 min. at 140°C.

In general, the tensile strip type of test piece (Fig. 3) has been used in both the cut growth and fatigue experiments.

For the cut growth tests, the rectangular strips were between 1 and 3 cm. in width and the length between the grips was not less than four times the width, so that their central portions were in simple extension. In the fatigue tests it was essential that failure did not occur at the grips and, to avoid this, dumbbell test pieces (B.S. 903, A.2; types C and D) were used. The dumbbells were die-stamped from the sheets and 2.5 cm. gauge lengths marked on the central parallel-sided sections, which were in simple extension when the test piece was stretched.

A tensile load–deflection test was carried out on at least one test piece from each molded sheet, the bench marks being in the central portions. From this, the stress–strain curve ($\sigma$–e) and hence, by graphical integration, the strain energy–strain curve (W–e) were obtained. It has been found that for most unfilled vulcanizates the stress–strain curve is not altered by repeated cyclic deformation, provided that allowance is made for the small amount of set which occurs.

The dynamic tests were all made within the frequency range of 100–130 cycles/min., the results not being sensitive to this small change in frequency.

For the cut growth tests, cuts about $^1/_2$ mm. in length were initiated with a razor blade in the center of one edge of the test piece, care being taken to ensure as nearly as possible that the cut was perpendicular to the edge and that the tip was normal to the major surface. Measurements of the cut length $c$ were made by using a microscope containing an eyepiece scale and with the test piece slightly strained to facilitate observation of the tip.

With the above test piece, the rate of growth $dc/dn$ and tearing energy $T$ vary with cut length, the latter in accord with eq. (4). In the results $dc/dn$ is approximated by $\Delta c/\Delta n$, were $\Delta c$ is the change in cut length measured over an interval of $\Delta n$ cycles. The corresponding $T$ value is taken as that at the average cut length in the interval $\Delta n$. Provided $\Delta c/c$ is less than 0.2, the error introduced by using $\Delta c/\Delta n$ for $dc/dn$ is estimated to be less than 1%. The accuracy of the $\Delta c$ measurement was always better than 10%.

## Cut Growth Behavior

Results of cut growth tests on tensile strips (Fig. 3) are shown in the form $dc/dn$ verses $T$ on logarithmic scales in Figure 5. Results applicable to the initial smooth growth from the razor cut have been omitted. The experimental points are reasonably well represented by the straight line, which has a slope of 2 in accord with eq. (3). The value of the cut growth constant $G$ is $20 \times 10^{16}$ c.g.s. units.

Also shown in Figure 5 are the results of tests carried out on pure shear (Fig. 2) and trousers type (Fig. 1) test pieces of the same vulcanizate. It is clear that the same cut growth relationship, eq. (3), is applicable to the three different types of test piece, thus confirming that the cut growth characteristics obtained are a property of the material and independent of the type of test.

Experimentally it was virtually impossible to obtain accurate results at tearing energies below those shown in Figure 5 for pure shear or trousers type test pieces. The tensile strip results extend a decade lower but are still above their lowest practicable limit. Other tests on tensile strips have shown that $T$ values of less than 0.01 kg./cm. are a practical possibility, but the cut growth rates were so slow that the cut lengths were only measured about once a month.

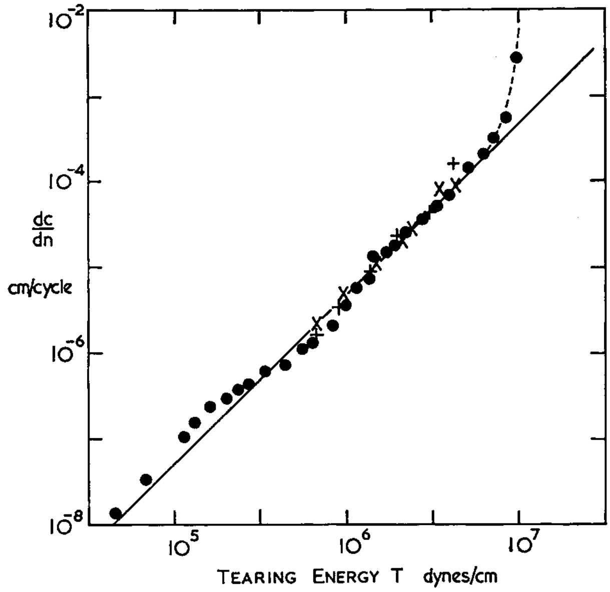


Fig. 5. Relationship between the rate of cut growth $dc/dn$ and the tearing energy $T$: (●) tensile strips; (×) trousers test pieces; (+) pure shear test pieces. The full line has a slope of 2 and corresponds to a $G$ value of $20 \times 10^{16}$ c.g.s. units.

The upward departure of the experimental points at high $T$ values (indicated by the broken line in Fig. 5) is probably due to the incipient catastrophic tearing discussed earlier, where the cut growth becomes time-dependent.

It was found experimentally by Rivlin and Thomas[1] that, provided the cut length $c$ did not exceed about one-fifth of the test piece width, the extension ratio in the bulk of the test piece was substantially unchanged by the presence of the cut, and eq. (4) remained applicable. This limitation on cut length was observed in the results shown in Figure 5.

The tensile cut growth test pieces, from which most of the data of Figure 5 were obtained, were cycled until failure occurred by rupture of the strip. The numbers of cycles to failure $N$ are tabulated in Table I, together with the maximum extension ratio $\lambda$ and the initial cut length $c_0$. Since $2kW$ can be found from the measured value of $\lambda$, the fatigue life $N$ can be calculated (Table I) from eq. (7), the $G$ value of $20 \times 10^{16}$ c.g.s. units found from the cut growth measurements being used. Although the tests cover a range of $c_0$ of 5:1, $2kW$ of 70:1, and $N$ of 3000:1 the greatest variation in calculated and observed values of $N$ is 1.5:1 at low $N$, the difference being less than 20% at higher values of $N$.

The cut growth relationship, eq. (3), and the cut growth constant $G$ could have been obtained from any suitable test piece as shown by the consistency of the results given in Figure 5, but it was experimentally more convenient to use tensile strips in the present case.

TABLE I
Calculated and Observed Fatigue Lives

| | | $N$, kcycles | |
|---|---|---|---|
| λ | $c_0$, cm. × $10^3$ | Observed | Calculated |
| 2.36 | 36 | 3.0 | 4.4 |
| 2.345 | 43 | 2.7 | 3.8 |
| 2.10 | 18 | 14 | 18 |
| 2.10 | 60 | 3.9 | 5.3 |
| 2.08 | 46 | 6.5 | 7.2 |
| 1.85 | 46 | 12 | 14 |
| 1.83 | 53 | 12 | 13 |
| 1.49 | 104 | 32 | 31 |
| 1.48 | 80 | 46 | 45 |
| 1.47 | 43 | 78 | 88 |
| 1.46 | 98 | 34 | 42 |
| 1.36 | 42 | 200 | 210 |
| 1.26 | 43 | 530 | 560 |
| 1.205 | 70 | 700 | 760 |
| 1.173 | 69 | 1200 | 1400 |
| 1.125 | 104 | 2300 | 2400 |
| 1.098 | 78 | 8600 | 9500 |

## Growth of a Single Crack

The cut growth relationship, eq. (3), predicts a linear dependence between the number of cycles $n$ and the reciprocal of the corresponding cut length $c$ [eq. (6)]. In Figure 6 the results of one of the cut growth tests are plotted in this form, $c^{-1}$ versus $n$. The broken line represents eq. (6) in which the values of $2kW$ and $c_0$ appropriate to the test have been used and $G = 20 \times 10^{16}$ c.g.s. units.

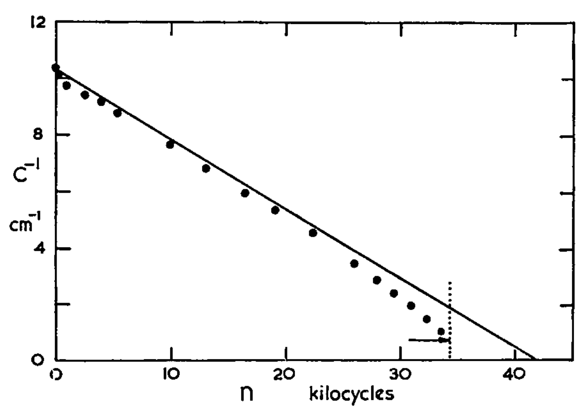


Fig. 6. Growth of a single cut in a tensile strip. The full line corresponds to eq. (6) or a value of $G$ of 20 × $10^{16}$ c.g.s. units determined from Figure 5. Failure occurred at the dotted line. The arrow corresponds to $T_c = 10^7$ c.g.s. units.

Initially the growth from the razor cut is smooth, but roughens after a few hundred cycles. This is not particularly clear in Figure 6, but other tests have shown that initially the cut growth constant $G_s$ is about $2 \times 10^{16}$ c.g.s. units.

The subsequent cut growth is maintained for most of the life of the test piece, the experimental points being parallel to the predicted relationship. The difference is due to the initial smooth growth and, on this test, is about 1.4 kcycles.

At high $T$ values the cut growth rate is greater than that predicted by eq. (3) (indicated in Fig. 5 by the broken line) and this is manifested in Figure 6 by the downward divergence of the points.

The tearing energy to which the experimental points in Figure 5 become asymptotic is denoted by $T_c$, the critical tearing energy, which for the rubber used in the present investigation is about $10^7$ dynes/cm. When the cut reaches a length corresponding to this critical value of tearing energy failure occurs catastrophically. This is illustrated in Figure 6, where the observed fatigue life is shown by a dotted line, and the cut length at which final catastrophic failure occurs (indicated by the arrow) is equivalent to this $T_c$ value of $10^7$ dynes/cm.

## Fatigue of Dumbbell Test Pieces

Fatigue lives of die-stamped dumbbell test pieces have been determined for test pieces relaxed completely during each cycle. The results are shown in Figure 7 as the number of cycles to failure $N$ plotted against the maximum extension ratio $\lambda$. If failure originates from small flaws, eq. (7) would be expected to apply, and $N$ should be proportional to $(2kW)^{-2}$.

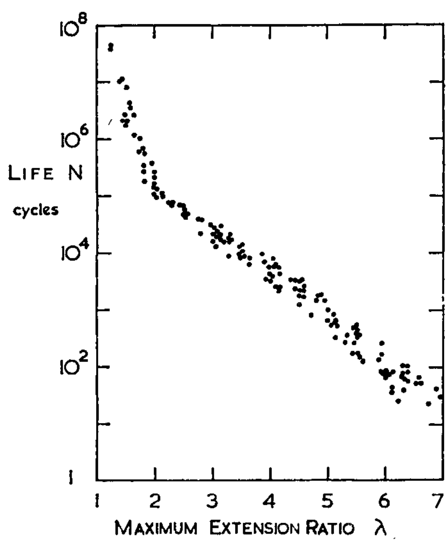


Fig. 7. Fatigue life $N$ as a function of the extension ratio $\lambda$.

This is seen to be substantially so in Figure 8, where the results have been replotted as $N$ versus $2kW$ on double logarithmic scales, the full line having the theoretical slope of $-2$.

Growth from the flaws in the die-stamped test pieces should follow a similar pattern to that of the single crack described in the previous section where it was stated that the rough growth was maintained for most of the life of the test piece. In fatigue tests the periods of smooth growth, initially while the tip of the flaw becomes roughened and latterly prior to catastrophic failure, are insignificant at low strains compared with the rough growth period. Under these conditions, below about 400% maximum strain, eq. (7) is adequate for determining $N$, as is shown in

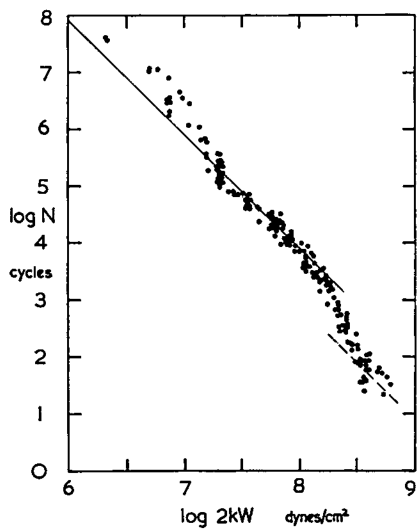


Fig. 8. Fatigue life $N$ as a function of the strain energy $2kW$. The lines have slopes of $-2$.

Figure 8. At higher strains, however, the period of rough growth is diminished and that of smooth growth predominates, resulting, in Figure 8, in the transition of the experimental points from the theoretical line, eq. (7), based on the rough cut growth constant $G$ to the broken line, in which $G_s$ replaces $G$.

The results are therefore consistent with fatigue failure being essentially a cut growth process taking place from small flaws or stress raisers present in the test piece. An estimate of their effective initial size, $c_0$, is $2.5 \times 10^{-3}$ cm., the value necessary for eq. (7) to correlate with the full line of Figure 8. The visual appearance of the cut edges of die-stamped test pieces is consistent with the presence of irregularities of this order.

Cracks are not easily visible to the naked eye until they are about ten times the above magnitude. Equation (6) predicts that flaws will not have grown to this extent until about 90% of the life has elapsed. This explains the observation that visible deterioration of rubber components is not normally apparent until failure is imminent.

The points of failure of the test pieces werer andomly distributed along the narrow central region, indicating that the die-stamping action did not introduce an isolated large flaw. Smaller, but clearly visible cuts were present in each test piece remote from the point of failure. The majority of these were at the die-stamped edges, but some were in the molded surfaces; in fact, failure has been observed to occur occasionally from flaws in these surfaces. Thus it appears that molded surfaces contain flaws or stress raisers having an effective size little less than those in the cut edges. Other stress raisers may be local inhomogeneities caused by hard particles or small regions of abnormal cross-linking density. Cracks formed by chemical attack, notably that of ozone, can also be sources of failure.

We thank Mr. G. J. Lake and Mr. R. L. Mawer for experimental assistance. This work forms part of the research program of The Natural Rubber Producers' Research Association.

## References

1. Rivlin, R. S., and A. G. Thomas, *J. Polymer Sci.*, **10**, 291 (1953).
2. Thomas, A. G., *J. Polymer Sci.*, **31**, 467 (1958).
3. Greensmith, H. W., *J. Appl. Polymer Sci.*, **7**, 993 (1963).

## Résumé

On a étudié la relation qui existe entre la croissance d'une coupure et la cassure à l'usure du caoutchouc naturel vulcanisé sous l'action répétée d'une charge. L'évolution de la croissance de la coupure a été étudiée en utilisant différents échantillons. Les résultats se sont avérés positifs lorsqu'on les interprète d'après la théorie de "l'énergie de rupture" qui a été développée précédemment. Les données les plus compréhensives ont été obtenues en mesurant la croissance d'une petite coupure faite sur le bord d'un échantillon qui subit des extensions simples. On a trouvé que la crossance de la coupure pour une extension est approximativement proportionnelle au carré de l'énergie de rupture maximum obtenue pendant l'extension. On peut déduire, grâce à cette relation, la durée de vie d'un échantillon ayant une petite coupure, a partir de la théorie d'élasticité, en tant que fonction de la grandeur de la coupure initiale et de la force maximum. Les résultats expérimentaux concordent bien avec la théorie. Une dépendance similaire de la force se retrouve dans le durée de vie d'échantillons sous forme de haltères dans lesquels on n'a pas produit intentionnellement de coupures; ceci concorde avec le mécanisme suivant lequel une cassure est due à la croissance de petites coupures présentes dans les échantillons. Leur grandeur effective peut être estimée aux environs de $2 \times 10^{-3}$ cm, ce qui correspond aux imperfections observées sur la surface des bords d'une coupure.

## Zusammenfassung

Die Beziehung zwischen Schnittwachstum und Ermüdungsverhalten von Naturkautschukvulkanisaten bei wiederholter Belastung wird untersucht. Das Verhalten beim Schnittwachstum wurde an mehreren Typen von Teststücken untersucht und die Ergebnisse lassen sich auf Grund des früher entwickelten Konzepts der "Reissenergie" einheitlich darstellen. Die umfassendsten Ergebnisse wurden durch Messung des Wachstums eines kleinen Schnittes in der Kante eines Streifens bei cyklischer, einfach

Dehnung erhalten. Das Schnittwachstum pro Cyklus ist ungefähr dem Quadrat des maximalen, während eines Cyklus auftretenden Reissenergie proportional. Mit dieser Beziehung kann die Ermüdungsbeständigkeit einer Probe mit einem kleinen Schnitt aus der Elastizitätstheorie als Funktion der Schnittlänge und der maximalen Dehnung abgeleitet werden. Die Versuchsergebnisse stimmen mit der Theorie gut überein. Eine ähnliche Dehnungsabhängigkeit wird für die Ermüdungsbeständigkeit gestanzter, hantelförmiger Teststücke ohne absichtlich erzeugte Fehlstellen gefunden; da entspricht einem Risswachstum kleiner in der Probe vorhandener Fehlstellen als Ermüdungsmechanismus. Ihre Grösse wird auf etwa $2 \times 10^{-3}$ cm geschätzt, was mit den beobachteten Oberflächenunregelmässigkeiten der Schneidekanten vereinbar ist.

Received October 22, 1962